Effect of the Quadrature Component in Single Sideband Transmission

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A PREVIOUS article [1] gives an analysis of single sideband transmission. Since that article was written this subject, particularly in its application to picture transmission and television, has assumed considerable importance. For this reason it now seems desirable to amplify the previous theoretical treatment and to indicate certain experimental results which have been obtained in the meantime. The present article gives experimental evidence that, for a given bandwidth, single sideband transmission is distinctly superior to double sideband in picture transmission.[2] It also gives a theoretical discussion which indicates that this is not inconsistent with the observed fact that oscillograms with single sideband transmission show considerable distortion.

As described in the previous article distortion to be considered in single sideband transmission as compared with double sideband transmission arises in three ways.

1. There may be present a slowly varying in-phase component due principally to the inaccurate location of the carrier frequency with respect to the edge of the filter characteristic.

2. The edge of the filter characteristic where the carrier is located may be so designed that there is a net distortion due to failure of the vestigial sideband to be accurately complementary to the principal sideband.

3. There is present a quadrature component which results in considerable distortion of the envelope of the received wave under ordinary conditions.

By in-phase component is meant a component whose carrier is in phase with the steady state carrier; by quadrature component is meant a component whose carrier is in quadrature with the steady state carrier. In some of the theoretical work in the present article, idealized

[1] *Trans. A. I. E. E.*, Vol. 47, p. 617, April 1928.

[2] A paper by Goldman: "Television Detail and Selective-Sideband Transmission," *Proc. I. R. E.*, Vol. 27, pp. 725–732, Nov. 1939, dealing with the same subject, has been published since our manuscript was sent to the printer. While the two papers reach similar conclusions there is considerable difference in method between them.

transducers have been assumed such that the first two effects listed above are absent. In the physical networks which are covered by the experimental work and part of the theoretical work these effects, while not absent, are found to be unimportant. The present discussion therefore is principally concerned with the third of these effects, namely, the quadrature component.

In a recent paper Smith, Trevor and Carter [3] have studied, both mathematically and experimentally, the matter of single sideband transmission over a rather simple filter and have found that the envelope is greatly distorted when the single sideband transmission is used. They give characteristics of their filters and also the location of the carrier frequencies so that it is possible to deduce that the first two effects, listed above, are unimportant for some of the carrier frequencies used. Their filter characteristics fall easily within the usual requirements for single sideband picture transmission at a speed appropriate to the bandwidth. Substantially the sole source of distortion in their work is the presence of the quadrature component, when the carrier frequency is suitably located.

Studies have also been made of a picture transmitting system of the type described by Reynolds.[4] This system makes use of single sideband transmission which had been found in previous experiments to be practicable. These previous experiments had shown that the quadrature component was present and was of considerable magnitude, but that the impairment in the picture was rather slight. They had also shown that if sufficient current was transmitted for the darkest portion of the picture the impairment could be reduced to the point where it was practically not detectable, and that a fairly small dark current would suffice.

Computations

The present section will be devoted to the computations of in-phase and quadrature components corresponding to certain assumed idealized characteristics, and reasons will be indicated why the picture impairment should be materially less than might be expected from the appearance of oscillographic records of the signal.

Figure 1 indicates the magnitude of the transfer admittance characteristic which will be assumed. The characteristic is made up of two half-cycles of a sine wave separated by a horizontal portion. The phase shift vs. frequency characteristic is a straight line. In order to simplify subsequent sketches this constant delay has been put equal

[3] *R.C.A. Review*, Vol. 3; p. 213, October 1938.
[4] *Bell System Technical Journal*, Vol. 15; p. 549, October 1936.

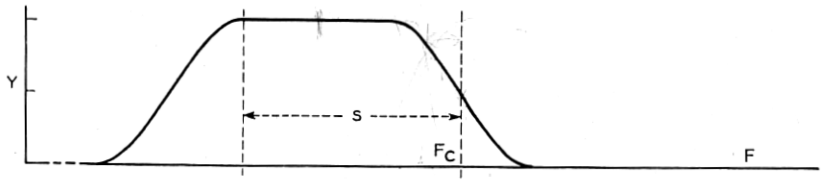

Fig. 1—Idealized transfer admittance characteristic. (Band pass system with no delay distortion; F_c is the carrier and s the fundamental dotting frequency.)

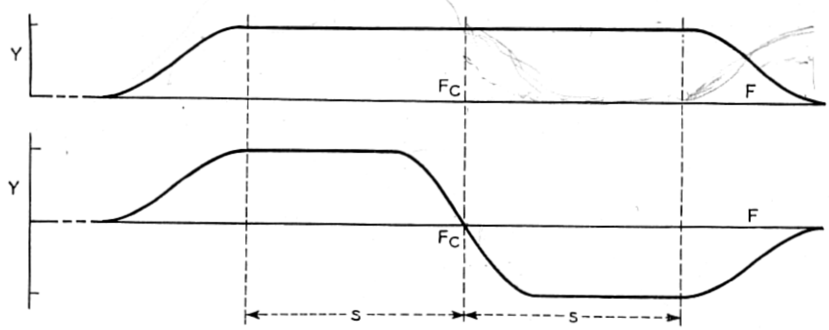

Fig. 2—Graphical analysis of transmission characteristic. (The sum of these characteristics equals that in Fig. 1. The upper gives received signals with carrier in phase, and the lower, in quadrature with the sent wave.)

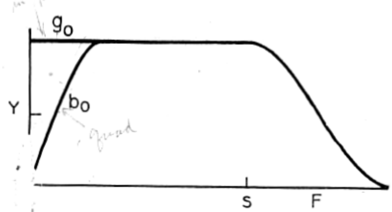

Fig. 3—Equivalent low-pass filter characteristic. (Used in computing envelopes of received signals for single side-band transmission.)

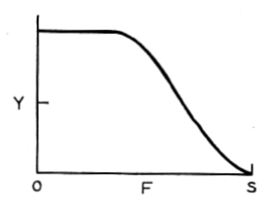

Fig. 4—Equivalent low-pass filter characteristic. (Used in computing envelopes of received signals with mid-band carrier.)

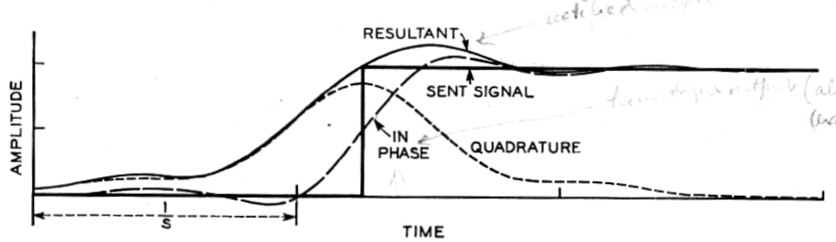

Fig. 5—Envelopes of received wave components for single transition, characteristic as shown in Fig. 1.

to zero. To take account of any constant delay it is sufficient to displace the computed curve by an amount equal to the delay. The characteristic of Fig. 1 may be separated into two components as indicated in Fig. 2, where the top one gives rise to the in-phase component and the bottom one to the quadrature component. F_c is the carrier frequency for single sideband computations, and it is assumed that F_c is great in comparison with the bandwidth. The characteristic of Fig. 1 does not differ greatly from those used in the experimental work. The quadrature component with the assumed characteristic is somewhat more pronounced than with the experimental ones. Figure 3 shows the equivalent low pass characteristics. Curve g_0 gives rise to the in-phase component and curve b_0 to the quadrature component. Figure 4 shows the low-pass characteristic which is equivalent to the original characteristic for double sideband computations with the carrier located in the middle.

Figure 5 gives the computed envelope for a single transition when this transducer is used on a single sideband basis. The figure shows the rectangular sent wave, the envelope of the in-phase component, the envelope of the quadrature component, and the envelope of the resultant wave. Figure 6 shows the corresponding received wave for the double sideband case. There is no quadrature component and the in-phase component and the resultant are identical. Figures 7 and 8 show the single sideband envelopes for a unit dot and a unit space, respectively. Figure 9 shows two dots in succession. Figure 10 shows the same case as Fig. 9 with the exception that dark current 14 db below the maximum current has been added. Figures 11 and 12 correspond to Figs. 9 and 10, the difference being that the dots are shorter. Figure 13 shows a succession of five dots. Figure 14 shows two dots as transmitted on a double sideband basis. In all the figures but 11 and 12 the fundamental dotting frequency is s as indicated in Figs. 1, 3 and 4. In Figs. 11 and 12 the dotting frequency is $4s/3$.

In comparing these figures a number of things will be apparent. In the first place, there is in the single sideband case a considerable broadening of all the marks due to the presence of the quadrature component. A second effect to be noted is that this broadening does not cause the dots to run together nearly as much as might be expected. This is particularly striking in Fig. 11 where the running together of the two dots is only slightly greater than it would be with the in-phase component alone. The reason for this is that when the dots tend to run together the contributions from successive dots to the quadrature component tend to cancel each other instead of adding to each other as is the case with in-phase components. The broadening of Fig. 11

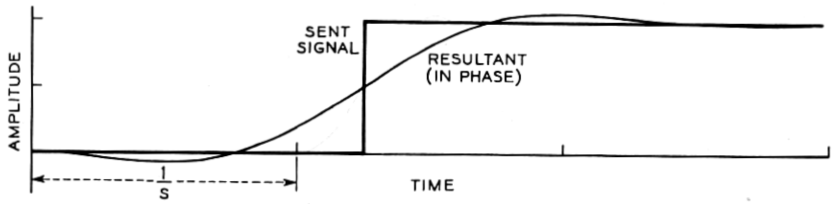

Fig. 6—Transmission of single reversal, carrier at mid-band.

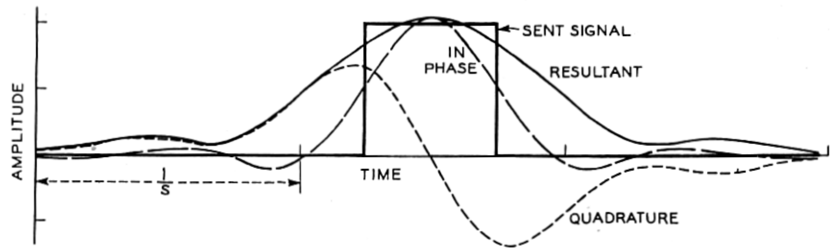

Fig. 7—Received signal for single dot, characteristic as in Fig. 1.

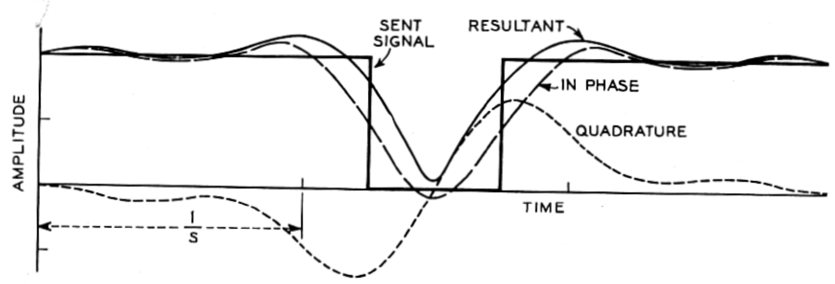

Fig. 8—Received signal for single space, characteristic as in Fig. 1.

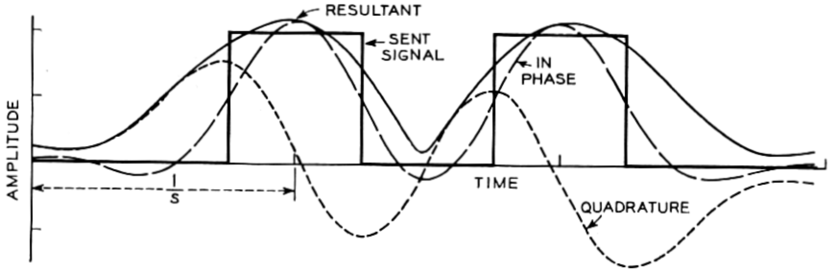

Fig. 9—Received signal for two dots in succession, characteristic as in Fig. 1.

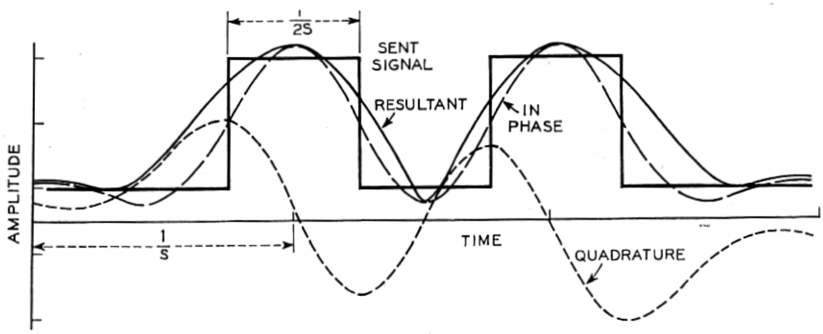

Fig. 10—Effect of transmitting dark current 14 db below maximum. (Compare with Fig. 9.)

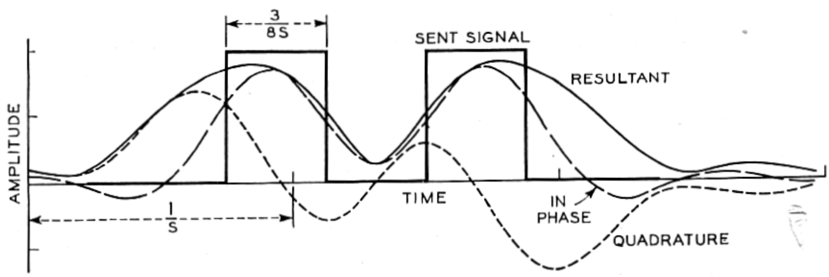

Fig. 11—Effect of shortening dots. (Compare with Fig. 9.)

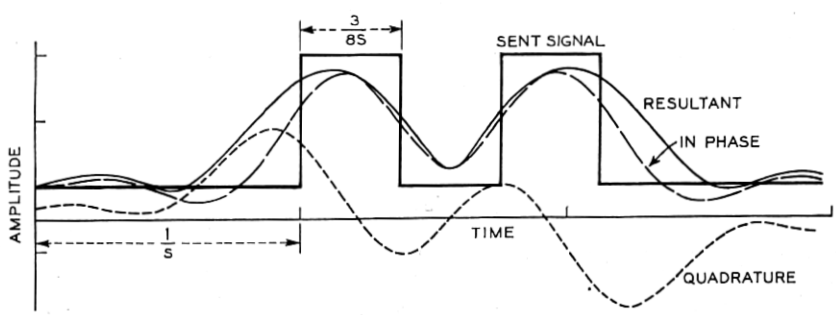

Fig. 12—Effect of adding dark current with shortened dots. (Compare with Figs. 11 and 10.)

and similar figures is principally on the outside of dots rather than on the inside. This tendency of the quadrature component to disappear when very short marks are employed, accounts for the observed fact that fine details are separated with single sideband methods as well as with double sideband methods using twice the bandwidth. Thirdly, the figures illustrate the effect of having finite dark current. This

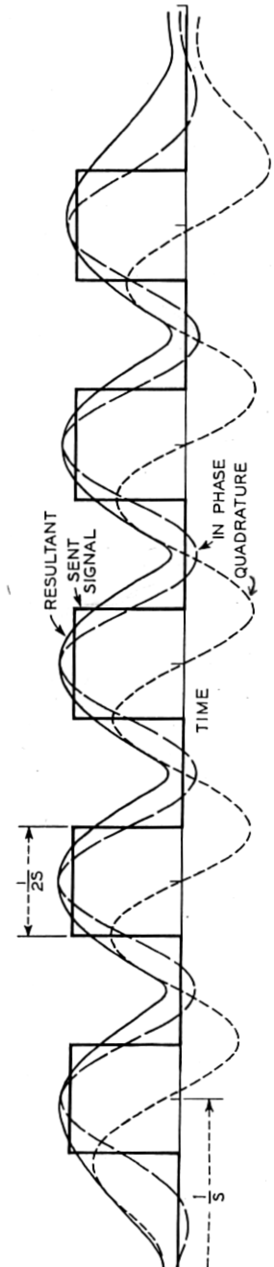

Fig. 13—Received signal for five dots, carrier as in Fig. 1.

effect is discussed below. Observations of transmitted pictures have shown that with the dark current of the magnitude indicated, it is practically impossible to detect the impairment from the quadrature component, although distortion is still evident on the computed curves. Figure 14 shows the relatively greater tendency for the double sideband dots to run together than the single sideband ones, for the same total bandwidth. The contributions from the two dots are, of course, in phase and therefore tend to add in the intervening space. It has been pointed out that with single sideband transmission the corresponding contributions to the quadrature component tend to cancel each other under these conditions.

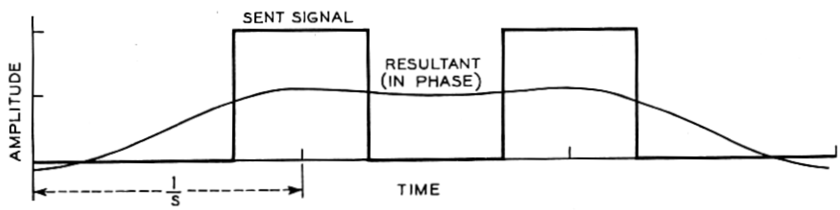

Fig. 14—Received signal for two dots, mid-band carrier.

Discussion

In estimating the effect of the quadrature component it is instructive to compare the in-phase component and the resultant in, say, Fig. 13. It will be evident that if the latter wave were used, for instance, for telegraph transmission there would be a considerable bias due to the quadrature component whereas the in-phase component shows practically no bias. Such a resultant wave would show a decided impairment unless steps were taken to counteract this bias.

If, however, the same figure is considered from the standpoint of picture transmission it will be clear that the difference is not nearly so striking. An obvious difference between a picture obtained with the in-phase component and one obtained with the resultant is that there is a tendency for a background of light gray to be present in the latter. Secondly, there is less contrast between the blacks and the whites. Both of these effects tend to be eliminated in photographic processes which follow the reception. Moreover, when they are not thus eliminated, they are not readily seen on examining the picture.

The presence of dark current increases the magnitude of the in-phase component as compared to the quadrature component. Since the resultant is equal to the r.m.s. value of these two components, it follows that increasing one component as compared to the other causes the larger component to approach the resultant. Consequently, adding the dark current causes the resultant to become more like the in-phase component, thus reducing distortion due to the quadrature component.

In half-tone pictures many of the transitions are in small steps. The quadrature component for small steps is frequently small compared to the total in-phase component. By reasoning similar to that in the previous paragraph, it follows that distortion due to quadrature component at small steps, is apt to be negligible. The quadrature effect in half-tones is also reduced by the fact that some of the changes are gradual.

The aperture effect has not been mentioned explicitly above. The aperture effect may be considered as being equivalent to a certain frequency characteristic and it may be assumed that the filter characteristics shown, include it. Incidentally, it is found that the aperture does not greatly affect the relationship between the in-phase and quadrature components.

While it may be expected that the quadrature component should have similar effects in picture transmission and in television, it is perhaps desirable to point out that there are important points of difference such as the presence of motion in the television images and the difference in response characteristics of a television screen and a photographic surface. It is not therefore an inevitable conclusion that television images will be as little affected as picture transmission images by the quadrature component.

Experimental

The conclusions are confirmed by certain experimental transmissions which were made over a picture machine employing a single sideband system as described by Reynolds.[5] The system makes use of 100 lines to the inch and has a total bandwidth of about 1000 cycles. The speed of the spot of light over the picture is about 20 inches per second. Two specimens of printing of different sizes were transmitted. A portion of each specimen one centimeter wide, after transmission, is shown in Fig. 15 enlarged to about five times its original size in order to avoid interference between the half-tone pattern and the picture pattern. Figure 15 should be viewed at about five times the normal reading distance. Group (a) was transmitted on a single sideband basis with the dark current reduced practically to zero. Group (b) was similarly transmitted, excepting that the dark current was 14 db below the maximum current. Group (c) shows a double sideband

[5] Loc. cit.

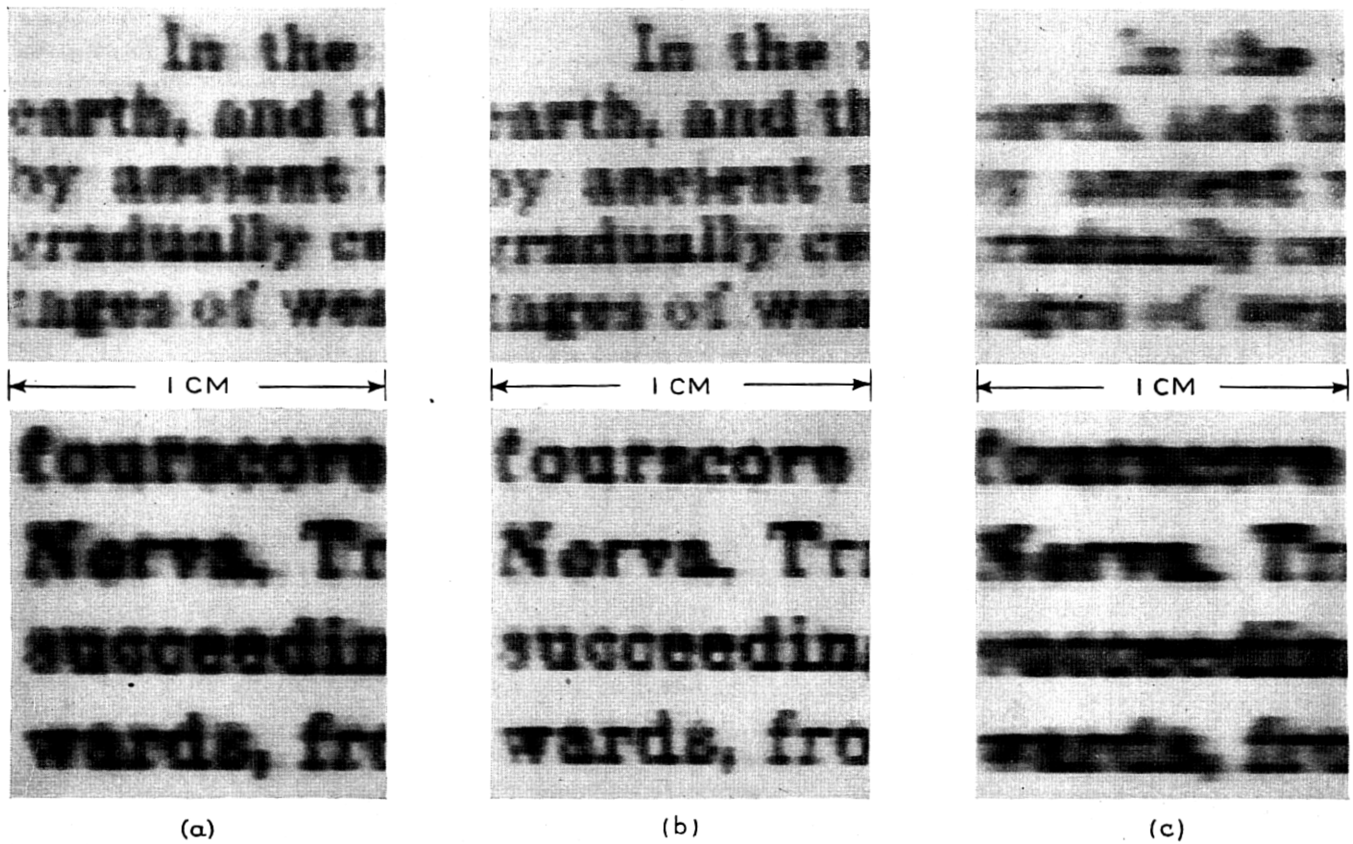

Fig. 15—Enlargements of transmitted printing. (*a*) Carrier at edge of band, dark current negligible. (*b*) Carrier at edge of band, dark current 14 db below maximum. (*c*) Carrier at center of band, dark current negligible. [The same transducer was used for (*a*), (*b*) and (*c*).]

transmission over the same transducer, the carrier being located at the center of the characteristic, the dark current being practically zero. It will be observed that the single sideband transmission gives materially more detail than the double sideband transmission, thus indicating that the presence of the quadrature component is not nearly so serious as a halving of the frequency range. It might perhaps be thought that this unfavorable showing of the double sideband transmission is due to the presence of some special distortion which might be expected in a filter designed for single sideband transmission when used in a manner not intended. On examining the characteristics no such distortion is found.

Acknowledgment

We have had the helpful cooperation of Dr. P. Mertz in this investigation.